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ABSTRACTS

OF PAPERS PRESENTED AT THE

SHERWOOD THEORY MEETING

March 12 - 13, 1973

Sponsored by

The Center for Plasma Physics and Thermonuclear Research

University of Texas

Austin, Texas 78712

For the

United States

Atomic Energy Commission

Washington, D.C.

SCHEDULE

1973 SHERWOOD THEORY MEETING

SUNDAY, 11 March

Registration at Villa Capri Motor Hotel

MONDAY, 12 March

8:30	AM	Morning Session Begins - Auditorium Thompson Conference Center
10:15	AM	Coffee Break
10:30	AM	Meeting Reconvenes
12:00	PM	Lunch - Dining Room Thompson Conference Center
1:30	PM	Afternoon Session Begins
3:00	PM	Coffee Break
3:15	PM	Meeting Reconvenes
5:30	PM	Afternoon Session Adjourns
6:30	PM	Cocktail Hour - Dining Room Thompson Conference Center
7:30	PM	Dinner - Dining Room Thompson Conference Center

TUESDAY, 13 March

8:30	AM	Morning Session Begins
10:15	AM	Coffee Break
10:30	AM	Meeting Reconvenes
12:00	AM	Lunch - Dining Room Thompson Conference Center
1:30	PM	Afternoon Session Begins
3:00	PM	Coffee Break
3:15	PM	Meeting Reconvenes
4:45	PM	Afternoon Session Adjourns

1973 SHERWOOD THEORY MEETING

Monday, March 12

Session A

8:30 AM — Chairman: A. A. Ware

8:30 - 10:05 Princeton Plasma Physics Laboratory Program

A1 P. H. Rutherford
"On Stability of the Elliptical Cross Section for a Free-Boundary Tokamak"

A2 A. H. Glasser and E. A. Frieman
"Stabilization of the Collisionless Trapped Particle Instability by Shaping of the Tokamak Cross Section"

A3 S. Yoshikawa
"On Toroidal and Helical Equilibria of Plasmas"

A4 J. M. Greene and R. Grimm
"Ion-Acoustic and Sound Waves"

A5 H. Okuda and J. M. Dawson
"Convective Cells, Drift Instabilities and Turbulence in a Plasma in a Magnetic Field"

A6 R. M. Kulsrud
"The Relation Between the Equilibria of Scyllacs and Stellarators"

10:00 - 10:15 Coffee

10:15 - 10:50 — Chairman: M. N. Rosenbluth

A7 R. Pozzoli and B. Coppi
"Three Electron Population Model for Current Carrying Plasmas"

A8 B. V. Waddell and B. Coppi
"Effects of Toroidal Modes on Particle Diffusion"

10:50 Texas Program

A9 A. A. Ware
"A Universal Ohms Law for Tokamak-Type Plasmas"

A10 R. D. Hazeltine and F. L. Hinton
"Collision Dominated Plasma Transport in Toroidal Confinement Systems"

A11 J. C. Wiley and F. L. Hinton
"Neoclassical Equilibria"

A12 D. W. Ross
"The Trapped Ion Instability in the Princeton Large Torus"

A13 W. Horton
"Diffusion from Low Frequency Fluctuations in a Straight Tokamak Model"

Session B

1:30 PM Chairman: B. Coppi

1:30 - 1:45

B1 D. J. Sigmar and J. F. Clarke
"Neoclassical Transport Theory of Multispecies Plasmas"

1:45 - 2:00

B2 M. N. Rosenbluth
"Nonlinear Stability of the Kink Mode"

2:00 - 2:15

B3 D. Dobrott and M. S. Chu
"MHD Stability of a Constant Pressure Doublet"

2:15 - 2:30

B4 H. R. Strauss
"Elliptic Tokamaks"

2:30 - 2:45

B5 A. B. Macmahon and A. A. Ware
"Steady State Solutions of the Neoclassical Transport Equations"

2:45 - 3:00

B6 W. B. Thompson
"Wave Propagation through Non-uniform Plasmas Some Points of Interest for R.F. Heating"

3:00 - 3:15 Coffee

3:15 PM Chairman: J. P. Freidberg

3:15 - 4:15 New York University Program

B7 H. Weitzner
"Survey of the NYU Program and Comments on Toroidal Equilibria"

B8 D. Stevens
"Simulation of the ATC Experiment"

B9 P. H. Sakanaka and J. P. Goedbloed
"Spectral Properties of the Diffuse Linear Pinch"

B10 J. A. Tataronis
"Consequences of the Continuous Spectrum in Ideal MHD"

B11 G. Spies
"Non-Existence of Stable Equilibria in Non-Linear Ideal MHD"

4:15 - 4:30

B12 E. P. Lee
"Evolution of a Relativistic Beam in a Torus"

4:30 - 4:45

B13 A. Mondelli and E. Ott
"Toroidal Plasma Equilibria with a Relativistic Component of Circulating Electrons"

4:45 - 5:00

B14 H. V. Wong, M. L. Sloan, J. R. Thompson, A. T. Drobott
"Stability of an Unneutralized Rigidly Rotating Electron Beam"

5:00 - 5:15

B15 R. N. Sudan
"Flute Instability of a Relativistic Beam Propagating in an Ionized Medium"

5:15 - 5:30

B16 L. E. Thode and R. N. Sudan
"Two-Stream Instability Heating of Plasma by Relativistic Electron Beams"

1973 SHERWOOD THEORY MEETING

Tuesday, March 13

Session C

8:30 AM — Chairman: R. N. Sudan

8:30 - 10:00 Lawrence Livermore Laboratory Program

C1 J. A. Byers, J. P. Holdren, A. B. Langdon, J. Killeen, A. A. Mirin, M. E. Rensink and C. G. Tull
"Review of Results from Astron Computational Models"

C2 H. L. Berk and D. E. Baldwin
"Wave Reflection in Mirror Machines"

C3 D. E. Baldwin and H. L. Berk
"Sensitivity of the Drift-Cone Mode to Orbit Diffusion"

C4 J. R. Hiskes and A. H. Futch
"Some Observations on Mirror Losses"

C5 B. McNamara and D. Fuss
"Toroidally Linked Minimum-B Mirror Fields Having -V""

C6 H. L. Berk and C. W. Hartman
"Stability of Hot Electron Plasma"

C7 J. Dibiase and J. Killeen
"Time Dependent Resistive Instability Calculations"

C8 W. A. Newcomb
"Drift Shock"

10:00 - 10:15 Coffee

10:15 AM — Chairman: W. B. Thompson

10:15 - 10:30

C9 B. D. Fried, G. Schmidt, and R. W. Gould
"Spatial Growth Characteristics of Parametric and Backscattering Instabilities"

10:30 - 10:45

C10 Y. C. Lee and C. S. Liu
"Rigorous Theory of Dielectric Function in a Magnetized Plasma"

10:45 - 11:00

C11 D. Choi and W. Horton
"Modified Kadomstev Spectrum from Renormalized Plasma Turbulence Theory"

11:00 - 11:15

C12 H. V. Wong
"Non-Linear Evolution of the Cross-field Electron-Ion Streaming Instability"

11:15 - 11:30

C13 O. Eldridge
"Do High Frequency Flute Modes Exist?"

11:30 - 11:45

C14 G. Joyce, D. Montgomery and M. Emery
"Electric Field Correlations in the Guiding-Center Plasma"

11:45 - 12:00

C15 D. Montgomery, G. Joyce and M. Emery
"Response of a Strongly-Magnetized Two-Dimensional Plasma to an External Electric Field"

12:00 - 12:15

C16 P. C. Liewer and N. A. Krall
"Results of a Self-Consistent Approach to Anomalous Resistivity"

Session D

1:30 PM Chairman: J. L. Johnson

1:30 - 3:00 Los Alamos Scientific Laboratory Program

1. J. P. Freidberg
"The Syllac Program"

2. F. A. Haas
"Kink Instabilities in a High β Tokamak"

3. D. A. Baker
"The Z Pinch Program"

4. R. L. Morse
"The Laser Fusion Program"

Abstracts for LASL Programs:

D1 J. P. Freidberg
"Stability of a Finite β, $\ell = 2$ Stellarator"

D2 B. M. Marder
"Screw Pinches with Elliptical Cross Sections"

D3 J. P. Freidberg and B. M. Marder
"Stability of Diffuse Two-Dimensional MHD Equilibria"

D4 J. P. Freidberg, B. M. Marder and H. Weitzner
"Wall Stabilization in $\ell = 0$ and $\ell = 1$ Systems"

D5 J. P. Freidberg and R. E. Siemon
"Flux-Surfaces for the Scyllac Configuration"

D6 J. P. Freidberg and F. A. Haas
"Kink Instabilities in a High-β Tokamak"

D7 D. A. Baker
"Review of Current Los Alamos Theoretical Work Related to Toroidal Z Pinch and Belt Pinch Equilibria Having Diffuse Profiles"

D8 J. P. Freidberg and H. R. Lewis
"Linear Stability Analysis of High-Beta Plasmas"

D9 C. W. Nielson and E. L. Lindman
"An Implicit, Two-Dimensional, Electromagnetic Plasma Simulation Code"

D10 W. P. Gula
"Simulation of a Straight Theta Pinch"

D11 T. A. Oliphant
"Pulsed θ-Pinch Reactor Burn Code"

D12 T. A. Oliphant
"Flushing and Refueling of a Pulsed Thermonuclear Reactor by Means of a Neutral Gas Layer"

3:00 - 3:15 Coffee

3:15 PM Chairman: D. E. Baldwin

3:15 - 4:45 Oak Ridge National Laboratory Program

D13 G. Guest
"Review of the ORNL Program in Plasma Theory"

D14 J. T. Hogan and R. A. Dory
"Tokamak Transport Theory at Oak Ridge National Laboratory"

D15 J. A. Rome, J. D. Callen and J. F. Clarke
"Theory of Neutral Beam Injection Into a Tokamak"

D16 D. B. Nelson and G. O. Spies
"MHD Stability of Toroidal Equilibria"

On Stability of the Elliptical Cross Section for a Free-Boundary Tokamak*

Paul H. Rutherford
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

ABSTRACT

When the plasma has a free surface, and the equilibrium is formed by suitable externally-produced shaping fields, an interesting question arises as to the MHD stability of a tokamak with elliptical cross section against changes of cross-sectional shape, e.g., more or less ellipticity ($m = 2$), triangulation ($m = 3$), etc. These modes are axisymmetric in the torus, and have $k_z = 0$ in the cylindrical approximation employed here. Due to the incompressibility constraint, these modes are <u>not</u> pure modes in a confocal system. For vertically elongated ellipses ($b > a$), besides instability to vertical displacement ($m = 1$), we find instability to triangulation ($m = 3$) if $b/a > 4.5$, but stability to $m = 2, 4$ for any b/a.

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* Work supported by U. S. AEC Contract AT(11-1)-3073.

Stabilization of the Collisionless Trapped Particle Instability by Shaping of the Tokamak Cross Section*

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ABSTRACT

Distortion of the cross section of a tokamak has been suggested as a means of improving ohmic heating and raising β.[1,2] We have investigated the effects of such distortion on the collisionless, electrostatic trapped particle mode and find that under certain conditions a maximum J configuration[3] can be approached. Results will be presented for finite elliptic and triangular distortions.

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* Work supported by U.S. AEC Contract AT(11-1)-3073.

[1] L. S. Soloviev, et al., Nuclear Fusion Supp., Pt. 1, (I.A.E.A., Vienna, 1969) p. 25.

[2] J. Callen, et al., in Plasma Physics and Controlled Nuclear Fusion Research (I.A.E.A., Vienna, 1971), Vol. II, p. 451.

[3] M. N. Rosenbluth, Phys. Fluids **11**, 869 (1968).

On Toroidal and Helical Equilibria of Plasmas *

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ABSTRACT

Several classes of toroidal and helical equilibria of arbitrary minor cross sections and/or arbitrary current distributions which have probably not been known previously, were found, in large aspect ratio expansion. Alternatively, exact toroidal equilibria function can be constructed with $\psi = A(\rho^2 - 1)^2 + B\rho^2 z^2 + C\rho^2 \log \rho + Dz^2$ for the constant current solution. Here A, B, C, and D are constants. The applications of this analysis, such as toroidal E-layer, good curvature trapping tokamak, etc., will be given.

*Work supported by USAEC Contract AT(11-1)-3073.

Ion-Acoustic and Sound Waves*

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ABSTRACT

We have studied the dispersion relation for ion waves in an infinite homogeneous medium, including the effects of electrons and of collisions. We have used the BGK collision model, in particular, a corrected and linearized form of the generalization given by Morse,[1] that includes the effects of ion-electron scattering. The inadequacies of this model for small collision frequency will be discussed. The resulting dispersion relation can be expressed as a 6×6 determinant involving combinations of the plasma dispersion relation. This is conveniently solved with the aid of an on-line root finding routine. The present work generalizes Varma and Bhadra,[2] by the inclusion of electron effects, including such things as electron Landau damping and heat conductivity. Of particular interest is finding appropriate models for the electrons in different collisional regimes.

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* Work supported by U. S. AEC Contract AT(11-1)-3073.

[1] T. F. Morse, Phys. Fluids 7, 2012 (1964).

[2] R. K. Varma and D. Bhadra, Phys. Fluids 7, 1091 (1964).

Convective Cells, Drift Instabilities and Turbulence in a Plasma in a Magnetic Field*

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ABSTRACT

Several extensions have been made on the enhanced diffusion due to thermally excited convective cells.[1] We find analytically and by computer simulation that for two dimensions, in addition to the classical ($D_\perp \sim B^{-2}$) plateau ($D_\perp \sim B^\circ$) and Bohm ($D_\perp \sim B^{-1}$) region, there is a new region between the classical and the plateau regions where $D_\perp$ increases with B ($D_\perp \sim B^1$ or B^2). This region is important for large values of m_i/m_e. Similar effects seem to be observed on Wendelstein II.[2] A more complete computer study has been made on the damping of the thermal vortices due to turbulence in two dimensions. It is confirmed that some existing turbulence theories are inadequate.[3] The difficulty appears to arise from inproper treatment of conservation of momentum and suggests the theory must be modified. A series of 2-, and 2 1/2-dimensional simulations were carried out to investigate the effect of ∇n on convective plasma diffusion. It is confirmed that the convective cells ($k_\parallel \approx 0$, $\omega \approx 0$) goes over to a finite gyro radius ion drift mode ($k_\parallel \approx 0$, $\omega \approx \omega^*_{ion}$) which results in a modest reduction in the convective diffusion rate. However, because this model is always unstable with respect to ion-ion collisions,[4] the amplitude of the convective motion increases with time. An increase in the diffusion rate with time is observed in the simulations which may be associated with this instability. At late times the enhanced diffusion rate becomes comparable to or may exceed that of $\nabla n = 0$ case.

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* Work supported by U.S. AEC Contract AT(11-1)-3073.

[1] H. Okuda and J. Dawson, Phys. Rev. Letters 28, 1625 (1972).

[2] G. Grieger et al., Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Vienna, Austria, 1971) Vol. III, p. 37.

[3] C. T. Dum and R.N. Sudan, Phys. Rev. Letters 23, 1149 (1969).

[4] A. A. Rukhadze and V. P. Silin, Sov. Phys. Dokl. 11, 606 (1967).

The Relation Between the Equilibria of Scyllacs and Stellarators*

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ABSTRACT

There are two independent asymptotic developments of toroidal equilibria in the literature one corresponding to the stellarator[1] and another corresponding to scyllac.[2] The shift of the plasma is in opposite directions and the origin of this difference is not clear.[3] It is shown that the two calculations can be extended so that they overlap and the reason for the differences is discussed.

---

* Work supported by U.S. AEC Contract no. AT(11-1)-3073

[1] J. L. Johnson, J. M. Greene, and K. E. Weimer, Nuclear Fusion 2, 16 (1962).

[2] M. N. Rosenbluth, J. L. Johnson, J. M. Greene, and K. E. Weimer, Phys. Fluids 12, 726 (1969).

[3] H. Grad and H. Weitzner, Phys. Fluids 13, 1418 (1970).

THREE ELECTRON POPULATION MODEL FOR CURRENT CARRYING PLASMAS

R. Pozzoli* and B. Coppi

Massachusetts Institute of Technology, Cambridge, Mass. 02139

Abstract. Three typical electron populations are distinguished in a high temperature two dimensional toroidal plasma: the current carrying ones under the influence of an externally applied electric field, the circulating ones but not carrying appreciable current, and the trapped ones. A model is proposed in which the current carrying population has a beam-like distribution superimposed on the Maxwellian of the other electron populations. This distribution is intended to be a modification of the equilibrium distribution of the neoclassical theory in a regime of high electron drift velocities. Travelling modes, along the magnetic field lines, with frequencies larger than the transit frequency of thermal electrons are found to be unstable for a reasonable choice of the relevant parameters. The effects of these modes can be compared to those of electron-electron collisions, in the sense that they can transfer momentum from the current carrying electrons to the main body of the distribution. The trapped and barely circulating electrons are affected by standing modes with frequencies of the order of the average trapped electron bounce frequency, and producing "quasi-banana" orbits with radial amplitudes larger than those of the known "bananas". The collisional diffusion associated with these orbits may provide an explanation for the observed "pseudoclassical" diffusion of particles and thermal energy.

*On leave from Laboratorio di Fisica del Plasma del C.N.R., Milano, Italy

Effects of Toroidal Modes on Particle Diffusion*

by

B. V. Waddell and B. Coppi

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In the presence of a special class of electrostatic standing-wave modes,[1] it is shown that the radial particle diffusion can be larger than the one predicted by neoclassical theory. We consider, in particular, the case where a single mode is excited about a rational magnetic surface. Then, a considerable fraction of the trapped particles can resonate with the mode and have their orbits (quasi-bananas) radially amplified while remaining trapped in the magnetic wells. Most of the circulating particles remain circulating with their orbits essentially unaffected by the mode. The transition region between quasi-banana and circulating orbits is composed of orbits which are alternately trapped and circulating.[1] The results of an approximate orbit analysis are used to obtain a trapped particle distribution function to lowest order in the inverse aspect ratio and the ratio of the collision frequency to the quasi-bounce frequency. Then, the well-known distribution function for circulating particles is joined smoothly to the one for trapped particles across the new transition region. Finally, by adopting a variational principle and a Lorentz collision operator, a diffusion coefficient is obtained for the electrons.

1. B. Coppi and A. Taroni, Report LGI/72/7/E (C.N.E.N., Frascati-Rome, Italy, 1972) to appear in Plasma Physics.

*Supported in part by the U. S. Atomic Energy Commission.

A Universal Ohms Law for Tokamak-type Plasmas*

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ABSTRACT

If the classical transport equations of Braginskii (or Herdan and Liley) are used for a stable Tokamak plasma ($\partial/\partial_\phi \equiv 0$) with the appropriate high collision frequency, the ϕ component of the current at any point is given by

$$j_\phi = \sigma_\parallel \left| \vec{E} + \vec{v} \times \vec{B} + \frac{0.32 \tilde{Q}_e \times \vec{B}}{n k T_e} \right|_\phi = \sigma_\parallel \left(E_\phi + v_r B_\theta + \frac{0.32 \tilde{Q}_{er} B_\theta}{n k T_e} \right) \quad (1)$$

for $Z = 1$, where $\sigma_\parallel$ is the well known Spitzer conductivity parallel to $\vec{B}$, and $\vec{Q}_e$ is the electron thermal flux relative to the center of mass. For higher Z, the conductivity is, of course, proportional to 1/Z but the numerical factor 0.32 in the last term changes only slowly; it is .375 for Z=2 and .47 for $Z=\infty$. (Notice that the conductivity is isotropic when the Ohms Law is written in this form.) If this equation is averaged on a magnetic surface in the matter $2\pi \langle j_\phi \rangle \equiv \int_0^{2\pi} j_\phi (R/R_0) d\theta$ the form of the equation is preserved and

$$\langle j_\phi \rangle = \sigma_\parallel \left(E_{\phi_0} + \langle v_r \rangle B_{\theta_0} + 0.32 \frac{\langle \tilde{Q}_{er} \rangle B_{\theta_0}}{n k T_e} \right) \quad (2)$$

where $\langle v_r \rangle$ is the average radial diffusion velocity given by Pfirsch and Schluter and $\langle Q_{er} \rangle$ is the corresponding enhanced average electron energy flux.

For lower collision frequencies, the neoclassical transport equations apply; with suitable rearrangement the corresponding fluxes are found to satisfy an identical relationship. Hence, equation (2) is a universal equation for $\langle j_\phi \rangle$. The universal character is traced to the similarity in the first order Larmor radius approximations to the distribution function in each case, the Larmor radius being in $|\vec{B}|$ for the classical equation and in B_θ for neoclassical.

Equation (2) is particularly useful for the banana regim and part of the plateau regime adjacent to the banana regime, since to lowest order in r/R, $\langle j_\phi \rangle$ is the same as the area-average of j_ϕ between magnetic surfaces, the latter being the appropriate average needed to determine the experimentally measured current or for applying Maxwell's $\vec{\nabla} \times \vec{B}$ equation at lowest order in (r/R). This approximation no longer applies in the Pfirsch-Schluter regime, but in that case the local form of the universal equation can be used.

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*Work supported by U.S. Atomic Energy Commission Contract AT-(40-1)-3458.

Collision Dominated Plasma Transport in Toroidal Confinement Systems*

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ABSTRACT

The collisional regime of neoclassical transport theory is investigated, using a moment equation approach as well as a method based on the drift-kinetic equation. Allowing for both density and temperature gradients, and an externally induced toroidal electric field, we derive the transport coefficients describing particle and energy flux perpendicular to the magnetic field of an axisymmetric confinement system. We also derive relations between the parallel and perpendicular fluxes, which are exact in the collisional regime.

*Work supported by the U.S. Atomic Energy Commission Contract No. AT (40-1) 3458.

Neoclassical Equilibria*

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ABSTRACT

Steady state equilibrium solutions of the radial neoclassical transport equations, including the transition between the banana and plateau regimes, have been found. The numerical method of solution will be described, as will the relationships between the various parameters which determine the equilibria: the central density, central electron and ion temperature, and the electric field. Very small boundary values on density and temperatures were obtained in some cases. The dependence of the central electron temperature on the density boundary value was striking, a small reduction in the boundary value resulting in a large increase in central temperature. The thermal stability of these equilibria was tested, using a time-dependent code described previously. Results will be reported.

*Work supported by the U.S. Atomic Energy Commission Contract No. AT (40-1) 3458.

The Trapped Ion Instability in the Princeton Large Torus*

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ABSTRACT

Estimated of growth rates of the trapped ion instability in the Princeton Large Torus (PLT) indicate that this mode will be dangerous. Elementary predictions of the containment time[1] yield a lower bound of about 1 msec., which, if approached, would be disastrous. The radial structure effects discussed previously by N.T. Gladd and the author[2] yield an increase in the containment time of at most a factor of thirty. Thus, further studies of effects which would improve the stability or limit the amplitude of this mode are needed. Since the eigenmodes are radially extended, one possibility is to take into account shear in the toroidal drift velocity caused by a varying radial electric field. Preliminary calculations indicate that radial fields of the order expected in Tokamaks[3] will be insufficient to stabilize the mode.

1. B.B. Kadomtsev and O.P. Pogutse, in Reviews of Plasma Physics, Leontovich, M.A., ed., (Consultants Bureau, N.Y., Vol. 5, p.379ff.)

2. N.T. Gladd and D.W. Ross, Univ. of Texas Report CPPT No. 61, February 1973 (submitted to Phys. Fluids).

3. M.N. Rosenbluth, P.H. Rutherford, J.B. Taylor, E.A. Frieman, and L.M. Kovrizhnikh, in Plasma Physics and Controlled Fusion Research, Fourth Conference Proceedings, Madison, 1971 (IAEA, Vienna, 1971), Vol. I, p. 495.

*Work supported by the U.S. Atomic Energy Commission Contract No. AEC AT (40-1) 3458.

Diffusion from Low Frequency Fluctuations in a Straight Tokamak Model*

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ABSTRACT

The low frequency ($\omega < \omega_{ci}$) stability and anomalous diffusion of a nonuniform, current-carrying plasma of the tokamak type in a large aspect ratio model are studied from the drift kinetic equation including a Lorentz electron-ion collision operator. The stability of the system is controlled by the magnetic shear, and the collision frequency is assumed to be low or appropriate to the banana-plateau transition in the corresponding tokamak. Even at this low collision frequency low energy electrons with $v < (T_e/m_e)^{\frac{1}{2}} (a/R)^{3/8}$ contribute a significant collisional contribution to the growth rate while faster electrons form the Landau resonance. For shear above the critical shear the modes are stable and we calculate the enhanced level of steady state fluctuations. At subcritical shear, as is typical in the computer generated neoclassical equilibrium profiles, a spectrum of fluctuations developes, and we use the resonance broadening theory of plasma turbulence to obtain the magnitude and scaling of the enhanced diffusion. In particular, the development of the theory by Cook and Taylor[1] is applicable if one assumes the modes form a random fluctuating field with a normal distribution. At intermediate collision frequencies where the distribution is held close to Maxwellian we obtain $D_\perp \simeq (\rho_e/r_n)(cT_e/eB)\, S^{-\frac{1}{2}} [1 - (S/S_c)^{\frac{3}{2}}]$ for $\nu_e > \nu_o \doteq (v_i \rho_i^2/r_n^3)(m_i S/m_e)(T_e/T_i)^{\frac{3}{2}}$ and at lower collision frequencies $D \approx \nu_e r_n^2 (m_e/m_i S)^{\frac{3}{2}}$. The corresponding fluctuation levels are obtained and are bounded by $|\tilde{n}/n_o|^2 \lesssim (\rho_i/r_n)^2 (T_e/T_i)$ consistent with the free energy arguments.

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* Work supported by the U.S. Atomic Energy Commission Contract No. AT-(40-1)-3458.

1 I. Cook and J. B. Taylor, "Electric Field Fluctuations in Turbulent Plasmas", Culham Laboratory Report CLM-P 325, 1972.

Neoclassical Transport Theory of Multispecies Plasmas

D.J. Sigmar[†)] and J.F. Clarke[*)]

Abstract

The transport theory of plasmas consisting of electrons, protons and high Z-impurity ions in toroidal confinement is developed, in the "banana" regime of small collision frequencies. The drift kinetic equations are solved explicitly for the particle distribution functions to lowest order in the inverse aspect ratio, using the operator for like collisions given by Rosenbluth, Hazeltine and Hinton, and a related operator for collisions of heavy ions on protons derived by a mass ratio expansion of the Fokker Planck operator.

Quantitative expressions for the radial particle- and heat fluxes and the electric (bootstrap) current are obtained from the appropriate moments of these distributions functions, as a function of the impurity concentration. An expression for the radial electric field as a function of the density- and temperature gradients and the impurity concentration is derived in the proton restframe in the limit of small impurity concentration and in the impurity ion restframe in the opposite limit. The modification of the Ware effect and the electrical resistivity due to impurity ions is also given. The transport coefficients reduce to those of Rosenbluth, Hazeltine and Hinton in the impurity free limit.

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†) Massachusetts Institute of Technology, Room 37-391.

*) Oak Ridge National Laboratory

Nonlinear Stability of the Kink Mode*

M. N. Rosenbluth
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* Work supported by U. S. Atomic Energy Commission.

MHD Stability of a Constant Pressure Doublet

D. Dobrott and M. S. Chu

Gulf General Atomic Company
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Magnetohydrodynamic stability of a straight Doublet is investigated. The plasma current is concentrated at the plasma-vacuum interface and the internal pressure is constant. Collapse, kink and sausage instabilities are examined in the various ranges of longitudinal wavelength. The perturbation modes do not separate in Doublet's noncircular cross section. However, a rapidly convergent approximation scheme enables one to show that the Kruskal-Shafranov limit is modified and that the plasma is stabilized by wall proximity.

Work supported by the U.S. Atomic Energy Commission, Contract No. AT(04-3)-167, Project Agreement No. 38.

Elliptic Tokamaks*

H.R. Strauss
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ABSTRACT

A Tokamak with elliptic cross section can carry a higher current, and attain a higher temperature, than a Tokamak with circular cross section. If an elliptic Tokamak is adiabatically transformed to a circular one, it will have β_p of order b/a, the ratio of the axes of the ellipse. In both the elliptic and circular state, the vacuum magnetic surfaces have separatrices close to the plasma, which gives the possibility of an axisymmetric civertor. Some simple elliptical MHD equilibria are presented, showing how the current profile affects the shape of the magnetic surfaces in the plasma. The main problems are the instability of the ellipse to displacement along its major axis, and the skin current which is induced by the transformation from ellipse to circle.

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Steady State Solutions of the Neoclassical Transport Equations *

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ABSTRACT

The neoclassical transport equations of Rosenbluth, Hazeltine and Hinton have been solved for steady state conditions in which the trapped particle pinch effect balances the particle diffusion. By using the banana regime coefficients at all radii, (this introduces only small errors for steady state solutions), and the simplest possible boundary conditions, the solutions for a given aspect ratio are found to depend on the single parameter $\alpha^2 = a^2\omega^2_{pi}/c^2$, where a is the minor tube radius and ω_{pi} the ion plasma frequency for the central density. The results indicate (i) a permanent skin current at intermediate values of α^2, (ii) the importance of the trapped particle Ettingshausen effect, (iii) the relative unimportance in the steady state of trapped particle effects on the current and (iv) almost constant $\bar{\bar{\beta}}$ and $\tau_E c^2/\pi\bar{\sigma}a^2$. Item (iv) agrees with the Tokamak T-3 experimental results and the numerical values show no large anomaly in the electron thermal conductivity other than that caused by the increased collision frequency which is indicated by the observed averaged electrical conductivity $\bar{\sigma}$.

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Wave Propagation through Non-uniform Plasmas

Some points of interest for r.f. heating

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ABSTRACT

Are there resonance effects at low frequencies much below the ion cyclotron frequency that could significantly enhance the effectiveness of such processes as ion transit time heating? A magnetohydrodynamic calculation exhibits such a resonance, and a similar one is found in low frequency plasma waves.

The M.H.D. calculation discusses the propagation of waves across a slab, $n(x)$, $p(x)$, with a shearing and non-uniform magnetic field $[0, B_y(x), B_z(x)]$. For compressive waves, the W.K.B. solution has a phase $\int F(x)dx$, where

$$F^2 = \frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}{\omega^2-\omega_o^2}$$

with $\omega_1^2, \omega_2^2 = k^2 c_T^2 \left\{1 \pm \sqrt{1-4\alpha}\right\}$ and $\omega_o^2 = k^2 c_T^2 \alpha$

and $k^2 = k_x^2 + k_y^2$: $\alpha = \frac{c_A^2 c_s^2}{c_T^4} \cos^2\theta$: $\cos\theta = \hat{\underset{\sim}{k}} \cdot \underset{\sim}{b}$

$c_T^2 = c_A^2 + c_s^2$, c_A, c_s = Alfvén and sound speeds.

Waves propagate in the window $\omega_o^2 < \omega^2 < \omega_1^2$, and since ω_o, ω_1 are functions of x, may reach either a stop band or resonance, at either of which points the fields are large.

For low frequency plasma waves, the M.H.D. ordering of the small Larmor radius expansion of the Vlasov equation, yields after elimination of the electrostatic potential, an equation for the magnetic compression $\delta B/B$. It has a similar W.K.B. solution with

$$F^2 = -k_\perp^2 + \frac{k_\parallel^2 c_A^2 - \omega^2}{c_A^2[\beta\alpha - 1]}$$

where $$\alpha = \frac{\left[Z'_-\left(\frac{\omega}{k_\parallel, v_o}\right) - Z'_+\right]\left[\frac{T_\perp}{T_\parallel} Z'_- - \frac{T_\perp}{T_\parallel} Z'_+\right]}{\frac{T'_\parallel}{T^+_\parallel} Z'_+ + Z'_-} - \left[\frac{T_\perp}{T_\parallel} Z'_- + \frac{T_\perp}{T_\parallel} Z'_+\right]$$

Near points at which a long wave length, k_a, ion acoustic mode can propagate the denominator in $\alpha = k_a^2/k_D^2$ and can be $\simeq \beta$. Thus a resonance can occur also for low frequency plasma waves.

Use of such resonances could greatly enhance the effectiveness of low frequency heating.

SIMULATION OF THE ATC EXPERIMENT

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Results of a simulation of the ATC device will be presented and compared with experimental data. The model used in the simulation mimics the experimental set up closely with respect to magnetic fields imposed on the plasma. However in the model the plasma is assumed to have all its toroidal current flowing on its surface. The plasma current in the computed equilibrium after compression is smaller than the plasma current observed (in ATC) immediately after compression.

SPECTRAL PROPERTIES OF THE DIFFUSE LINEAR PINCH

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The spectral properties of the diffuse linear pinch are investigated by means of the equation of motion. This equation is non-singular and behaves Sturmian at the unstable side of the spectrum, permitting a straightforward numerical solution for unstable modes. Stability is redefined in a practical sense as the absence of unstable modes growing faster than a certain cut-off growth rate.

A number of stable or weakly unstable configurations is found, roughly divided into two categories, viz. θ-pinch like configurations ($B_\theta \ll B_z$) featuring small growth rates and z-pinch like configurations ($B_\theta \sim B_z$) permitting high values of β. Sharp and diffuse profiles are compared. The important question is investigated as to how far profiles may deviate from a defined stable one without leading to disastrous instabilities.

Suydam-unstable configurations give rise to non-localized $m = 1$ modes having appreciable growth rates. With increasing m the growth rates increase slightly and the modes become more localized, showing a weaker dependence on the wall position. For increasing values of n (the number of nodes of the eigenfunction) the localization also increases, but the growth rate decreases rapidly. Exponentially small growth rates are obtained if Suydam's criterion is only slightly violated.

Suydam's criterion has been generalized to include stability against quasi-kink instabilities. The obtained criterion shows a completely different shear stabilization for high-or low-m instabilities associated with interchange-or kink-like behavior. It has been applied as a guide in the search for stable configurations.

CONSEQUENCES OF THE CONTINUOUS SPECTRUM IN IDEAL MHD

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It is well known that the equations of motion of ideal MHD, when linearized about a diffuse sheet pinch or screw pinch equilibrium, have a continuous spectrum. The purpose of the present paper is to discuss the plasma behavior which is a consequence of this continuum.

Only the sheet pinch in plane geometry and the theta pinch in cylindrical geometry are treated here. We investigate the time behavior of the plasma by using the Laplace transform to solve the initial value problem. It is shown that no coherent oscillations are sustained in a diffuse plasma. One finds instead a slow decay and dispersion of the oscillations to zero due to phase mixing of the modes in the continuum. If the profile is steep, the decay is exponential for a finite time, and then changes to the classical relaxation rate of at least 1/t. The exponential damping is found by the proper analytic continuation of a dispersion relation.

Finally we show that the continuum implies energy absorption which, as a consequence, could lead to plasma heating. Expressions are developed which give the rate at which energy is absorbed by the plasma, and the conditions favorable to attaining strong absorption.

NON-EXISTENCE OF STABLE EQUILIBRIA IN NON-LINEAR IDEAL MHD

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It has been shown [1] that, according to the linearized MHD-equations, every magnetically confined equilibrium with well-defined smooth pressure-surfaces is subject to linearly growing instabilities, irrespective of the sign of δW (which yields information only about exponential growth). In the present note we show that this feature of the linearized theory has a counterpart in the non-linear theory. Using a variational definition of stability whose relevance rests on the assumption that any two states with the same values of certain constants of the motion are accessible from each other, we show that every equilibrium is unstable. Our physical interpretation is that every equilibrium state has arbitrarily close neighboring states which, when taken as initial data, yield solutions of the equations of motion that cannot be expected to stay close to the equilibrium state. The neighboring states for which we are able to make this plausible are exactly those which have been shown [1] to give rise to the linearly growing solutions of the linearized equations.

Reference: [1] H. Grad, Sherwood Theoretical Meeting
Los Alamos, 1972

EVOLUTION OF A RELATIVISTIC BEAM IN A TORUS*

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February 22, 1973

ABSTRACT

The approach to the force-free state of a beam in a torus is studied in the limit of large aspect ratio. The driving mechanism is the emission of synchrotron radiation, which is partially counter-acted by collisions with background particles.

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Prepared for presentation at the Annual CTR Theory Meeting, Austin, Texas, March 12-13, 1973.

Toroidal Plasma Equilibria with a Relativistic Component of Circulating Electrons

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We study analytically the equilibrium of an axisymmetric, large aspect-ratio, toroidal plasma which is enclosed by a perfectly conducting toroidal container. The new feature of the analysis is the presence of a relativistic electron beam which circulates around the torus thus producing a poloidal magnetic field. This problem may be of interest in connection with recent toroidal beam injection experiments at Cornell University, and tokamaks. The equilibria are obtained from the relativistic virial theorem. Specific examples are given wherein the plasma pressure is characterized by its dependence on the magnetic flux function and the relativistic electron beam is characterized by an electron distribution function which is a delta function in energy and canonical momentum in the toroidal direction.

Stability of an Unneutralized Rigidly Rotating Electron Beam

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ABSTRACT

An unneutralized rigidly rotating electron beam is absolutely stable if the distribution function is a monotonically decreasing function of $H_o - \omega_L L_z$ and if $\rho_B \omega_L / c < 1$, where H_o and L_z are the single particle energy and canonical angular momentum, ω_L is the angular velocity, and ρ_B is the radius of an enclosing infinitely conducting cylindrical wall.

Flute Instability of a Relativistic Beam Propagating in an Ionized Medium

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Experiments contemplating the use of high intensity relativistic electron beams to heat a plasma to temperatures of thermonuclear interest are planned or are underway in several laboratories. For the success of these experiments, it is important to ensure that the beam does not break up due to any gross macroscopic instability. When a beam is injected into a dense plasma, it is promptly neutralized electrostatically as well as magnetically by the plasma return current. The magnetic neutralization is almost complete when the beam radius is much larger than the electromagnetic skin depth c/ω_p. This situation viz counterflowing beam and plasma current is inherently unstable to a magnetic flute instability which tends to break up the beam by the repulsion of these currents that results in the flutes being driven radially outwards. A longitudinal guide field of magnitude such that the total beam energy density $n_b mc^2\gamma$ is less than the magnetic energy density $B^2/8\pi$, stabilizes these mode. A similar criterian applies for beams injected perpendicular to a static B field. There is experimental evidence for this instability.

Two-Stream Instability Heating of Plasma by Relativistic Electron Beams

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The homogeneous interaction between an intense relativistic electron beam and a dense plasma is investigated using an electrostatic particle code. The fraction of initial beam energy which is converted into electric field energy during the hydrodynamic two-stream instability as a function of $S \equiv \beta_o^2 \gamma_o \left(\frac{n_b}{2n_e}\right)^{1/3}$ is found to maximize at $S \sim .6$. Here n_b and n_e are beam and plasma electron densities respectively, $\beta_o = v_o/c$, $\gamma_o = (1 - \beta_o^2)^{-1/2}$, and v_o is the initial mean beam velocity. This effect is shown to be the result of an increased energy spread in the trapped electron distribution leading to an effective increase in the spread of trapping frequencies of these electrons. Within a few e-folding times before wave saturation, harmonics are generated due to spatial bunching and their energy spectrum has a power law dependence. The wave spectrum built up as a result of the two-stream instability decays via the oscillation's two-stream instability. Essentially all the energy lost by the beam goes into producing heavily populated, high energy tails on the electron distribution. The ions role is to provide an efficient coupling of energy transfer from large amplitude, high phase velocity waves to lower phase velocity waves which are Landau damped by the plasma electrons.

REVIEW OF RESULTS FROM ASTRON COMPUTATIONAL MODELS*

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February 26, 1973

ABSTRACT

For the last two years we have made an extensive effort at simulation of relativistic electron layers in Astron-like configurations, including such effects as resistor focussing and resistor energy loss. The following are the major results from SUPERLAYER, a 2-D (r,z) electromagnetic code. (1) Under charge neutralized conditions the field reversal, ζ, saturates due to an axial lengthening during the stacking of several pulses. This result was a major factor in the decision to turn the experiment to stacking in vacuum. (2) The axial dynamics of pulse focussing and pulse stacking has also been examined in fine detail by the 1-D code T-LAYER. Both codes show that vacuum pulse trapping and focussing depends sensitively on parameters such as resistor L/R time and the injection current. This is also observed in the experiment. It is possible to choose parameters such that a vacuum pulse will focus better than a charge neutral pulse. (3) Stacking in vacuum, if not first limited by the axial lengthening, ultimately runs into difficulty at sufficiently high ζ, in the range 0.5 to 1.0, due to increased radial expansion. (4) Sufficiently large current (only four times that available in the experiment and only two times that available with the split pulse) can produce field reversal in one pulse (charge neutral). These results indicate the possibility of achieving field reversal by stacking of two pulses with the present experimental equipment under ideal conditions. (5) For a wide variety of conditions the efficiency of single pulse trapping is nearly perfect. The experiment's poor trapping is then clearly due to nonideal effects such as r,θ instability or combined effects of poor beam quality and field asymmetries.

The results from an idealized r,θ code show that for a moderate ζ vacuum layer the negative mass instability will result in large radial loss and prevent buildup in vacuum to reversal.

*Work performed under the auspices of the U. S. Atomic Energy Agency.

WAVE REFLECTION IN MIRROR MACHINES*

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February 27, 1973

ABSTRACT

Stability of waves in mirror machines where $\omega_{pi}^2/\omega_{ci}^2 \lesssim 1$ depend critically on the wave reflection properties of the ends (instability occurs when the reflection is nearly unity). The reflection properties are sensitive to the shape of the density at the ends. We have shown that if the plasma does not fill the well, nearly total wave reflection can be expected above a critical density. A mechanism which will prevent the filling of the well is that in general a particle drift surface will be on lines of varying mirror ratio. Hence, the effective mirror ratio for a particle will be associated with the line of minimum mirror ratio on the drift surface. Hence, waves on the higher mirror ratio lines are more likely to be reflected. If external electric field drifts are comparable to the grad B drifts, the drift surfaces are altered, and there is likely to be a further spread in mirror ratios in a drift surface; thereby causing a stronger tendency towards instability. Baseball II is observed to be more unstable during its steady state operation with a neutral beam source than in its decay mode. We have estimated by rough dimensional arguments that in Baseball II the E x B drift caused by a neutral beam source is comparable to the ion grad B drift, and we are attempting to apply the above reflection mechanism to explain the observed stability characteristics.

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*Work performed under the auspices of the U.S. Atomic Energy Commission.

SENSITIVITY OF THE DRFIT-CONE MODE TO ORBIT DIFFUSION*

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ABSTRACT

A long standing theoretical question concerning the density decay of the 2XII mirror plasma has been the apparent absence or benignity of the electron drift-ion loss cone mode,[1] for which the plasma density exceeds its stability threshold by 10^2-10^3, and $k_\perp a_i$ becomes exceedingly large. Such effects as finite-β, magnetic field intensity and density variation, and magnetic line fanning have been shown to have stabilizing tendencies, but to not change the basic picture.

As a beginning to considering the effect of orbit diffusion upon the stability, we have included the parameters $D_e(D_i)$ for electron (ion) diffusion in the familiar ik^2D corrections to ω. If $k_\perp a_i > 1$, then $D_e/D_i \cong (k_\perp a_i)$, and the mechanism is most effective for large $k_\perp$. We find that $D_i/D_e \sim 1/7$ when $D_e \sim .1\ \omega_{ci} a_i^2$ is sufficient for complete stabilization of the mode at all $k_\perp$, with increases in D_e allowing decreases in D_i for marginal stability.

Possible sources of this diffusion might be vortex diffusion,[2] Rosenbluth-Post convective fluctuations,[3] or self-diffusion in the field spectrum of a drift-cone instability constrained to low amplitude. All such possibilities are under investigation.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] D. E. Baldwin, et al., Proc. IV Internation Conference on Plasma Physics and Controlled Nuclear Fusion Research, Vol. II, IAEA Vienna (1971),735.

[2] J. B. Taylor, Vth European Conference on Controlled Fusion and Plasma Physics, Vol. II, Grenoble (1972), 83.

[3] D. E. Baldwin and J. D. Callen, Phys. Rev. Letters 28, 1686 (1972).

SOME OBSERVATIONS ON MIRROR LOSSES*

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February 27, 1973

ABSTRACT

As a first step in estimating neutral beam requirements for future injection experiments, an analysis is made of loss rates from several mirror experiments. Data from BBII, PR-6, MSTSII, DECCAII, and 2XII permit an evaluation of loss rates over and above charge exchange loss rates. The observed rates are compared with classical, i.e., Fokker-Planck, rates to give a measure of the anomalous losses. An attempt is made to correlate these anomalous losses as a function of density or $\varepsilon = \omega_{pe}^2/\omega_{ce}^2$.

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*Work performed under the auspices of the U. S. Atomic Energy Agency.

TOROIDALLY LINKED MINIMUM-B MIRROR FIELDS HAVING -V"

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ABSTRACT

One solution to the end loss problem in mirror machines is to link several minimum-B fields around a torus. In doing so one would like to have an average magnetic well and a topologically stable field with shear and rotational transform. The analytic expressions for V", i, |B| on an arbitrary magnetic axis given by Solov'ev and Shafranov, Reviews of Plasma Physics, Vol. 5, have been programmed and tested on stellarator and helically symmetric fields. Basic field functions appropriate to a set of linked mirrors have been varied, through suitable parameters, to yield examples of the desired minimum-B, -V" fields.

The advantages of the configuration are (1) Mirror injection methods are natural for plasma production. (2) End losses are not a problem. (3) The field offers local and average stabilizng properties. (4) All trapped particles are trapped in regions of good curvature. (5) Plasma between mirrors is easily stabilized by short connection lengths. The code, TORAX, is a useful design tool for any closed non-axisymmetric field.

*Work performed under the auspices of the U. S. Atomic Energy Agency. Prepared for presentation at the Annual CTR Theory Meeting, Austin, Texas, March 12-13, 1973

STABILITY OF A HOT ELECTRON PLASMA*

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February 27, 1973

ABSTRACT

The stability of a hot electron plasma is studied where $T_e/T_i \gtrsim M/m$. In addition to the stability criteria obtained by Krall,[1] we find that instability can arise at frequencies comparable to the ion cyclotron frequency. These instabilities can be stabilized with a sufficient amount of cold plasma.

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*Work performed under the auspices of the U. S. Atomic Energy Agency.

[1]N. A. Krall, Phys. Fluids, 4, 820 (1966).

TIME DEPENDENT RESISTIVE INSTABILITY CALCULATIONS*

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ABSTRACT

A hydromagnetic model has been developed to study resistive instabilities in cylindrical geometry, and the model is applied to study specific diffuse pinch configurations. The MHD equations include the effects of compressibility, finite resistivity, viscosity and thermal conductivity. The plasma equilibrium configuration is assumed known and is specified by $B_{\theta 0}(r)$, $B_{z0}(r)$, $\eta_0(r)$, $\rho_0(r)$, $T_0(r)$ and these functions can be chosen to describe a particular experiment. Perturbations of the form $f_1(r,t)e^{i(m\theta+kz)}$ are used for all plasma and field variables, and the resulting linear partial differential equations are solved numerically as an initial value problem using an implicit difference scheme. A set of 6 equations for B_{r1}, $B_{\theta 1}$, T_1, v_{r1}, $v_{\theta 1}$, v_{z1} is obtained, and the calculation is started by specifying an initial perturbation. For a particular problem, the parameters m, k and $S = \tau_R/\tau_H$, the ratio of resistive diffusion time to hydromagnetic transit time must also be given. For certain cases an exponential growth occurs and the growth rate p(m,k,S) is calculated. The model can be used to test equilibrium configurations against both tearing and rippling modes. Extensive parametric studies for the m = 0, 1 modes have been done for the tearing mode using a Bessel function field model, and resistive instabilities were found where no ideal instabilities exist. The results show a strong dependence of p on k and S, with stability increasing as R_W decreases. The infinite conductivity case was also tested and the results agree well with analytic theory. The model can thus be applied to both MHD and resistive instabilities in cylindrical geometry.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

DRIFT SHOCK*

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ABSTRACT

An example is analyzed for shock formation by a nonlinear drift wave in a guiding-center plasma (slow-drift ordering).

* Work performed under the auspices of the U. S. Atomic Energy Commission.

Spatial Growth Characteristics of Parametric and Backscattering Instabilities. B.D. FRIED and G. SCHMIDT, UCLA and R.W. GOULD, California Inst. of Tech.--Both the familiar electrostatic parametric instability (photon → phonon plus plasmon) and the backscattering instability (photon → photon plus plasmon or phonon) are usually considered in the context of initial value problems (real k, complex ω). However, some applications of the theory, including plasma heating and laser backscattering, involve steady state excitation necessitating consideration of spatial growth for real ω. When damping effects, collisional or collisionless, are neglected, the decay instabilities are always absolutely unstable (exponential growth in time at a given location). For the choice of k which allows exact frequency match, the pump power threshold for instability is zero; for other k, there is a non-zero threshold; and above it the instability is always absolute. A simple analysis, virtually identical with that used for beam instabilities, shows that when damping effects are included, there is a non-zero threshold, P_c, for convective instability, and only above a still higher threshold, $P_A > P_c$, does the instability become absolute, a behavior reminiscent of loss cone instabilities. If the decay length (v_{group}/γ_{decay}) for one product wave greatly exceeds that of the other, $P_c/P_A \ll 1$ so there is convective growth for a large range of pump powers. It follows that wave propagation, with the attendant refraction and change of k, may, in many circumstances, be more important for saturation than nonlinear effects. For the purely growing modes (oscillating two stream instability in the parametric case, filamentation in the backscattering problem) there is no region of convective instability.

Rigorous Theory of Dielectric Function in a Magnetized Plasma.

Y.C. LEE, UCLA and C.S. LIU, Inst. for Advanced Study.--A general formulation for the dielectric function has been developed for plasmas in thermal equilibrium so that the particle orbit diffusion can be taken into account. The formulation can describe both the charged-neutral collisions and the diffusion caused by thermal fluctuations. Without assuming a thermal equilibrium system, a general relation for the correlation functions is obtained which reduces to a nonlinear fluctuation-dissipation theorem for thermal equilibrium plasmas. When a strongly-magnetized two-dimensional plasma is considered, it is shown that a low-frequency slightly-damped mode exists for $k < k_D$. Calculations of auto-correlation functions for the $\underset{\sim}{k}$th Fourier mode show a Gaussian-type decay for $k \gg k_D$ and a damped oscillation for $k \ll k_D$.

Modified Kadomstev Spectrum from Renormalized Plasma Turbulence Theory*

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ABSTRACT

From the collisionless Vlasov-Poisson equations the renormalization of plasma turbulence is derived by a selective summing of secular terms to all orders in perturbation theory. The resulting renormalized theory agrees with earlier renormalizations of quasilinear theory and, in the mode coupling equation, exhibits the same symmetries known in weak turbulence theory. Our formulation differs from that of Rudakov and Tsytovich by containing a separation of turbulent k-scales and by the associated feature of retaining the form of induced wave scattering in the nonresonant mode coupling terms. The theory is applied to the problem of finding the stationary spectrum for current driven ion-acoustic turbulence. In the renormalized theory induced wave scattering off the screened ions dominates in the regime $(m_e/m_i)^{\frac{1}{3}} \leqslant T_i/T_e \ll 1$ and the modified Kadomstev spectrum contains a low $k\lambda_D$-cut off self-consistently determined within the collisionless theory. The turbulent energy density and effective collision frequency are computed from the new spectrum.

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Non-linear Evolution of the Cross-field Electron-Ion Streaming Instability*

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ABSTRACT

An alternative model of the early time evolution of the cross-field electron-ion streaming instability[1,2] is proposed. The evolution proceeds in two stages, an initial rapidly growing stage corresponding to a coherent phase followed by a slower growing stage corresponding to a random phase in which the wave-particle correlation time is short compared to the diffusion time. The effect of the magnetic field on the electron motion diminishes as the diffusion time decreases below the electron cyclotron period[1]. For a one-dimensional spectrum of unstable waves, this transition is predicted to occur at higher levels of turbulence than that predicted by Lampe, et al[1].

1. M. Lampe, W.N. Manheimer, J.B. McBride, J.H. Orens, K. Papadopoulos, R. Shanny, and R.N. Sudan, Phys. Fluids 15, 662 (1972).

2. D.W. Forslund, R. Morse, C. Nielson and J. Fu, Phys. Fluids 15, 1303 (1972).

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DO HIGH FREQUENCY FLUTE MODES EXIST?

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There is a strong wave-particle interaction at resonant surfaces in a magnetized plasma, the surfaces where a wave frequency matches a multiple of the ion cyclotron frequency. An ion drifting through these surfaces exchanges energy with the wave and produces a local growth or damping of the mode. For modes with finite $k_{||}$ the energy exchange leads to Landau damping; but the interaction, and the damping, persists for flutelike modes with $k_{||} \simeq 0$. For high frequency modes of a plasma contained by a magnetic field with moderate spatial variation there are several resonant surfaces intersecting field lines. Wave energy must flow for between these sources and sinks and the mode must have some parallel wavenumber. For a cold plasma the interaction is always stabilizing.

ELECTRIC FIELD CORRELATIONS IN THE GUIDING-CENTER PLASMA

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A quantity of central interest in transport theory for the two-dimensional guiding-center plasma is the autocorrelation of the $\underset{\sim}{k}$th Fourier component of the electric field,

$$S_{\underset{\sim}{k}}(t) \equiv \langle \underset{\sim}{E}^*(\underset{\sim}{k},0) \cdot \underset{\sim}{E}(\underset{\sim}{k},t) \rangle \quad ,$$

where $\langle \; \rangle$ means a thermal equilibrium ensemble average. We have measured this quantity in numerical simulations involving 4000 particles and periodic boundary conditions. [Initial positions for the particles are generated from an unmagnetized, 2D, P.I.C. code, which is allowed to run through a few thermal relaxation times. During this time, a spectrum $\langle |\underset{\sim}{E}(\underset{\sim}{k})|^2 \rangle$ develops which approximates the thermal equilibrium spectrum obtained from an $(n_o \lambda_D{}^2)^{-1}$ expansion of the BBGKY hierarchy.]

The results (individual runs and averages over an "ensemble" of 8 runs) show a damped, non-oscillatory behavior for $S_{\underset{\sim}{k}}(t)$. $S_{\underset{\sim}{k}}(t)$ is well fit over most of its range by $S_{\underset{\sim}{k}}(0) \exp(-k^2R(t))$, where $R(t)$ is a monotonically increasing function which after a short initial time goes as $Dt/2$. D is, up to a factor of $\sqrt{2}$, the Taylor-McNamara diffusion coefficient. The absence of oscillatory behavior is at variance with the random-phase theory of Taylor and Thompson, which predicts a sinusoidal undamped oscillation for $S_{\underset{\sim}{k}}(t)$.

Fluctuations of the individual runs about the ensemble average are large.

[1]Supported in part by USAEC Grant AT(11-1)-2059.
[2]Supported in part by NASA Grant NGL-16-001-043.

RESPONSE OF A STRONGLY-MAGNETIZED TWO-DIMENSIONAL PLASMA TO AN EXTERNAL ELECTRIC FIELD

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The response of a two-dimensional electrostatic guiding center plasma to an external, uniform electric field $\underset{\sim}{E}_o$ is studied numerically. If one assumes an unbounded plasma, the response is a uniform translation with velocity $c\underset{\sim}{E}_o \times \underset{\sim}{B}/B^2$. But if boundaries exist (and it is unlikely that an extensive thermodynamic limit exists for the 2D guiding center plasma, anyway), uniform translation is no longer possible. The effect of a mechanically reflecting wall can be represented by a very strong force field $\underset{\sim}{F}$ which vanishes except in the immediate neighborhood of the wall, and is directed normally inward from it. A particle being forced against the wall by an electric field $\underset{\sim}{E}_o$ experiences a sign-dependent $\underset{\sim}{F} \times \underset{\sim}{B}$ drift (electrons one way, ions the other) parallel to the wall. The particles crawl along the wall until a collision with another particle returns them to the plasma or until the $\underset{\sim}{E}_o \times \underset{\sim}{B}$ drift lies in a sense to remove them from the wall.

These features occur in a preliminary 10,000-particle numerical experiment: (1) the effect of the electric field $\underset{\sim}{E}_o$ is to induce macroscopic vortices, in pairs, in the plasma; (2) the amount of energy in the vortices continues to increase with time; (3) the vortices develop more rapidly for larger values of $\underset{\sim}{E}_o$; and (4) there is apparently no local proportionality between $\underset{\sim}{E}_o$ and the electric current density $\underset{\sim}{j}$; $\underset{\sim}{j}$ and $\underset{\sim}{E}_o$ are not usually perpendicular or parallel. This last feature implies that an earlier formulation[3] of the problem in terms of a Kubo conductivity derived from weak-field perturbation theory is probably unsatisfactory.

[1]Supported in part by NASA Grant NGL-16-001-043.
[2]Supported in part by USAEC Grant AT(11-1)-2059.
[3]D. Montgomery and F. Tappert, Phys. Fluids 15, 683 (1972).

RESULTS OF A SELF-CONSISTENT APPROACH TO ANOMALOUS RESISTIVITY

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In this work we present a self-consistent model for turbulent resistivity and diffusion in plasmas which carry a current across a magnetic field. In particular, the time and space development of a magnetic pulse in a plasma is explored, emphasizing the fact that turbulent resistivity grows and damps as the plasma conditions change to make various plasma waves unstable or stable. Using local stability properties, selected modes grow exponentially, producing corresponding changes in plasma transport coefficients. These coefficients (resistivity-like terms) influence the plasma by broadening magnetic pulses and heating both the electrons and ions; this alters the local stability properties, which changes the resistivity itself. We present (1) a numerical model in which the resistivity is consistent with the time history of local stability properties; (2) a detailed comparison between the results of a self-consistent model and the results of previous approaches (classical resistivity, fixed anomalous resistivity, e.g., $\nu = \omega_{pi}$); and (3) a detailed comparison between the results of the model and the results of three types of fast θ-pinch implosion experiments.

STABILITY OF A FINITE β, $\ell = 2$ STELLARATOR *

by

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ABSTRACT

The stability of an infinitely long, high β, $\ell = 2$ stellarator has been investigated using the ideal magnetohydrodynamic sharp boundary model. The motivation for studying this problem was to determine whether the $\ell = 2$ configuration would have favorable stability properties in a parameter range of interest to the Scyllac program.

The results of this calculation are as follows: for m = 1, k = 0, a critical β is found as would be predicted from low β stellarator theory. For finite β,δ the value of β_{crit} is somewhat lower than predicted by the low β theory in the regime of experimental interest. In fact the highest value of β_{crit} for any strength $\ell = 2$ filed is about .17. This is somewhat discouraging from the Scyllac point of view because of the high β requirements of implosion heating. However, the stability picture is actually much worse. The reason for this is that as in low β stellarators, the worst modes are not k = 0 modes, but interchange modes in which the perturbation remains in phase with the rotational transform of the magnetic field. We have shown that if the wavenumber k of the perturbation satisfies

$$\frac{k}{h} = (2p + m)\left(1 - \frac{\iota}{2\pi}\right) - m \qquad |p| = 0, 1, 2 \ldots \quad ,$$

then the critical β is reduced to zero. Here h is the helical pitch number, m the mode number and $\iota/2\pi$ the rotational transform. Thus, if one allows long but not infinite wavelength m = 1 modes, wavenumbers k exist which are unstable for all β. Clearly this configuration does not, as it stands, possess any obvious advantage over the $\ell = 1$ Scyllac configuration.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

SCREW PINCHES WITH ELLIPTICAL CROSS SECTIONS*

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ABSTRACT

It was hoped that ellipticity in the cross section of a screw pinch would relax the equilibrium and stability criteria and allow much higher betas to be obtained. Such does not appear to be the case.

The straight elliptical screw pinch was treated using ideal MHD sharp boundary theory. The form δW was minimized by expanding the perturbation as a Fourier series, thus allowing mode coupling. The wave length of the perturbation in the Z direction was assumed to be much larger than the small radius of the plasma.

While the ellipticity helps achieve high β toroidal equilibria with q greater than 1, such configurations are no longer stable.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

STABILITY OF DIFFUSE TWO-DIMENSIONAL MHD EQUILIBRIA*

by
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ABSTRACT

We have developed a numerical method for computing the growth rates and eigenfunctions for an arbitrary two-dimensional, diffuse, high-β, magnetohydrodynamic equilibrium.

The object of the procedure is to minimize δW subject to the correct magnetohydrodynamic normalization and thus obtain the actual eigenvalues and eigenfunctions of the system. In the minimization we first Fourier transform the perturbation with respect to the ignorable coordinate of the equilibrium. The remaining part of the perturbation is expanded as double series of complete admissible functions of the equilibrium coordinates. The minimization is then performed over the expansion coefficients.

As a test of the procedure we treated the problem of the bumpy θ pinch with arbitrary size bumpiness. Our results are in excellent agreement with the existing diffuse theory of Weitzner in the limit of small bumpiness. As the bumpiness increased, it was found that the normalized growth rates increased only slightly; that is the normalized growth rate is insensitive to bump strength. Further, the sharp boundary wall stabilization effect is not found for diffuse profiles.

The numerical procedure turns out to (1) be easy to code, (2) require only moderate storage and (3) run quickly. In addition, by choosing appropriate expansion functions, there is considerable flexibility in applying boundary conditions.

At present the method appears to be applicable to a wide class of diffuse equilibria including, the finite β, $\ell = 1$ Scyllac, finite β stellarators, Tokamaks, and toroidal screw pinches.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

WALL STABILIZATION IN $\ell = 0$ and $\ell = 1$ SYSTEMS *

by

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The theory of the $\ell = 0$ bumpy pinch contains the well-known sharp boundary result of Haas and Wesson which predicts that the m = 1 mode could be stabilized if $\beta > 1/(1 + a^2/b^2)$. More recent studies by Weitzner and at Los Alamos showed that any diffuse profile must be unstable to an m = 1 mode. The basic source of the discrepancy has to do with the difference in the structure of the spectrum between sharp boundary and diffuse profiles. In the diffuse case, the unstable spectrum consists of an infinite but discrete set of modes whose eigenvalues form a point of accumulation at $\omega^2 = 0$. The most unstable mode has no nodes and is a true gross mode for sufficiently low β. The second worst mode has one node, the third worst mode, two nodes, etc., as would be found in a classical Sturm-Liouville problem. We emphasize that this structure pertains only for low β (i.e. less than required for wall stabilization). As the β is increased over the value where one would expect wall stabilization in the sharp boundary model, the diffuse spectrum changes dramatically. The gross mode with no nodes becomes stable, while the fastest growing mode now has one node. As one steepens the profile, all that remains intact is the gross no-node, mode. The growth rates, in both the low and high-β region, of the modes with higher nodal structure, decrease as the profile steepens, eventually approaching zero in the limit of the sharp boundary. It is this behavior of the spectrum which explains the apparent discrepancy between sharp and diffuse profiles in an $\ell = 0$ system.

A similar investigation was carried out for the diffuse $\ell = 1$ system. In this case the spectral picture was considerably simpler and in fact confirmed predictions of earlier Los Alamos work in which a simple scaling argument predicted that the stabilizing effect of the gross mode would scale as (a/b^4) as was found in sharp boundary theory. An examination of the $\ell = 1$ spectrum did indeed verify this behavior for the gross mode, and uncovered as well a whole host of other, more highly structured modes. These modes were discrete and their eigenvalues were all stable forming a point of accumulation at $\omega^2 = 0$. The eigenvalues of these modes were very insensitive to wall radius in contrast to the no-node gross mode.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

FLUX-SURFACES FOR THE SCYLLAC CONFIGURATION*

by

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ABSTRACT

The Scyllac high-β toroidal equilibrium is created by vacuum fields which to good approximation can be written

$$\vec{B} = B_o \vec{e}_z \left(1 - \frac{r}{R}\right) - \vec{\nabla}\phi$$

$$\phi = B_{\ell=1}\, 2\left[I_1(hr)/h\right] \sin\, (\theta - hz)$$

$$+ B_{\ell=0}\left[I_o(hr)/h\right] \sin\, (hz)$$

$$+ B_r\, r \sin\theta$$

$$+ B_{12}\, \left[I_1(2hr)/h\right] \sin\, (\theta - 2hz)$$

Here B_o is the large toroidal θ-pinch field, $B_{\ell=1}$ and $B_{\ell=0}$ are the amplitudes of helical fields which provide the basic interference force to oppose the toroidal gradient, and B_v and B_{12} are smaller helical fields included to reduce perturbations caused by the ℓ = 1,0 fields.

It was not clear what combinations of such fields, if any, would have flux surfaces. However, field lines followed numerically have appeared to remain on closed surfaces (within integration accuracy) for every combination of parameters used.

This surprising result has very significant consequences for the experiment since it provides a straight-forward procedure by which the desired vacuum fields can be generated. By machining the inner surface of a single-turn aluminum coil to match the desired flux surface, one can create the toroidal fields with the same coil that provides the large θ-pinch field. The resulting fields are far more free of unwanted components than fields produced by auxiliary current windings and extensive magnetic field measurements in such coils show complete agreement with the calculation.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

KINK INSTABILITIES IN A HIGH-β TOKAMAK*

by

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ABSTRACT

In this paper we consider the MHD stability of a toroidal, sharp-boundary, circular cross section plasma. All current is confined to a thin-skin layer and $\beta (= 2p/B_\phi^2)$ is of order ϵ, the inverse aspect ratio. The equilibrium is maintained by a vertical magnetic field and the condition that the poloidal flux-surfaces neighboring the plasma be closed leads to an equilibrium limit on β. Stability of this model is examined using the energy principle. For modes $n \neq 0$ (n is the mode number taken the long way) the minimizing perturbations are incompressible, and the potential energy can be cast into a form involving only the radial perturbation of the surface. Final minimization is carried through numerically for the modes $n = 1,2,3$ and the results presented as a set of marginal stability lines in a β v. q diagram, q being an appropriately defined "safety factor". For sufficiently low-β the worst mode, $n = 1$, is unstable for $q < 1$ and stable for $q > 1$, a result consistent with previous theory. However, a limiting value for β is observed ($\beta \simeq 0.2\ \epsilon$) above which the $n = 1$ mode is unstable for <u>all</u> q. Along the $n = 1$ marginal line $m = 2$ is found to be the dominant harmonic, m being the mode number taken the short way. The effect of conducting walls is also considered.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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REVIEW OF CURRENT LOS ALAMOS THEORETICAL WORK RELATED TO TOROIDAL Z PINCH AND BELT PINCH EQUILIBRIA HAVING DIFFUSE PROFILES*

by
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ABSTRACT

Numerical solution of the MHD equilibrium equations permits studies of realistic plasma profiles in toroidal geometry unhampered by the usual restrictions on boundary shapes, aspect ratio and beta. A combined analytic-numerical stability treatment of these two-dimensional equilibria has shown the existence of MHD stable toroidal Z-pinch configurations.

The effects of field diffusion on MHD stable configurations for the linear Z-pinch have been investigated. These studies demonstrate the existence of initially MHD stable profiles for which classical diffusion does not lead to the appearance of unstable gross modes on time scales which prevent the achievement of the Lawson Criterion.

Diffuse profile toroidal belt-pinch equilibria have been computed which show that small ellipticity contracted solutions exist as well as the elongated belt-like solutions for the same highly elongated boundary cross section. Numerical solutions have been obtained which represent belt-pinch equilibria with toroidal betas of 80% and with q values greater than unity on all flux surfaces. These equilibria have been studied for totally closed conducting boundaries and for open configurations having a separatrix similar to the Garching experiment.

A study of the Z-pinch susceptibility to short wavelength instabilities due to the large field aligned currents has been completed. The complete homogeneous electromagnetic hot plasma dispersion relation in the presence of field aligned currents was solved numerically to find the most unstable modes. For ion betas greater than .02 the critical drift velocity is approximately three times the ion thermal speed and is insensitive to the electron temperature. It was concluded that this instability is not important for the present Los Alamos experiment.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

LINEAR STABILITY ANALYSIS OF HIGH-BETA PLASMAS*

by

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ABSTRACT

The Vlasov-fluid model[1] was proposed as the starting point for a more realistic study of the stability of high-beta plasmas than is possible in terms of the usual ideal MHD approximation. In the Vlasov-fluid model the ions are treated as collisionless, the electrons are treated as a massless fluid, charge neutrality is assumed, and the displacement current in Maxwell's equations is neglected. Also, the equilibrium ion distribution function is required to be a function only of the total energy. With these assumptions, the allowed equilibrium configurations of the magnetic field, the criterion for marginal stability, and an estimate of the m = 1 growth rate for the case of a θ-pinch with a small azimuthal magnetic field are identical in the Vlasov-fluid model and ideal MHD. However, an estimate of the growth rate for m = 2 can be much smaller in the Vlasov-fluid model than in ideal MHD for a hot θ-pinch with a small azimuthal magnetic field. These estimates of m = 1 and m = 2 growth rates within the framework of the Vlasov-fluid model are in semi-quantitative agreement with experimental observations.

To enable numerical computation of growth rates using the Vlasov-fluid model for a wide range of equilibria, including diffuse profiles, a new formulation[2] of the linearized Vlasov-fluid model has been developed using ideas that were effective in a numerical treatment of one-dimensional electrostatic oscillations that was based on Hamilton's variational principle for collisionless plasmas.[3] A computer code has been written that applies this formulation to equilibria that are axially and translationally symmetric, and computations have been carried out for a sharp boundary equilibrium. For this example, analytic estimates of growth rates given in Ref. 1 have been verified, and it has been observed that growth rates are insensitive to truncation of expansions that are used in the new formulation. The expansions are in terms of eigenfunctions of the unperturbed Liouville operator and in terms of eigenfunctions of the incompressible MHD operator.

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AN IMPLICIT, TWO-DIMENSIONAL, ELECTROMAGNETIC PLASMA SIMULATION CODE*

by

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ABSTRACT

A plasma simulation code which combines desirable features of previous simulation codes with some useful new features has been written and tested. All six components of the electromagnetic field and all three particle velocity components are included. However, only two space dimensions are included due to the size and speed limitations of present computers. Spatial periodicity is assumed in one direction, but the other space direction may have boundary conditions determined periodically or by fixed field specification, with a propogating electromagnetic wave being a special case of the latter. Any density distribution, uniform in the periodic direction and piecewise linear in the other direction, may be initialized.

The field computation proceeds in terms of potentials rather than directly in terms of field quantities because this allows an implicit solution of the wave-equation which is free of the restriction on time step imposed by the Courant condition for explicit methods. It also allows a convenient technique to be used to correct for the finite-mesh distortion of the wave dispersion relation. Specifically the vector potential A satisfies the difference equation

$$\nabla^2(A^{n+1} + 2A^n + A^{n-1})/4$$
$$- (1 + (\alpha^2 D^2/4)\nabla^2)\quad (A^{n+1} - 2A^n + A^{n-1})/c^2 (\Delta t)^2$$
$$= (4\pi/c)\, J^n + \nabla X^n/c$$

where $1/D^2 = 1/(\Delta x)^2 + 1/(\Delta y)^2$ and where ∇^2 is the five point numerical form of the Laplace operator. The function X^n satisfies $\nabla^2 X^n = 4\pi\ (\nabla_1 \cdot J^n)$ in order to guarantee $\nabla \cdot E = 4\pi\ \rho$ as described by Morse and Nielson.[1] The proper choice of α^2 allows bringing the dispersion relation to a form close to that of the continuum if D is of the same order as Δt. The Laplace operators are inverted by a generalized Buneman algorithm which allows periodicity in one direction and arbitrary boundary conditions in the other. The fields are obtained from the potentials by the double width space derivative $(\partial A/\partial X)\ i = (A_{i+1} - A_{i-1})/(2\Delta x)$. This differencing, together with the definition of currents and charge on the same mesh as suggested by Boris and Lee[2], leads to a code with a single mesh system for all field and source quantities.

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2. J. P. Boris and R. Lee, NRL Memo. 2418, March, 1972.

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Simulation of a Straight Theta Pinch *

by

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Abstract

A modified particle-in-cell computer model has been used to simulate the post-compression phase of the 5-meter straight theta pinch at Los Alamos. The plasma is assumed to be axisymmetric and the ions are assumed to be tied to the field lines. The ion motion along the field lines is calculated from the equation

$$F = -\mu \, dB/dZ$$

where μ is the magnetic moment of the ion. The magnetic field is calculated from the pressure balance equation

$$\frac{B_o^2}{2\mu_o} = \frac{B^2}{2\mu_o} + P_\perp$$

where μ_o is the magnetic permeability of free space, B_o is the applied magnetic field, and $P_\perp$ is the ion and electron kinetic pressure perpendicular to the magnetic field.

The usual procedure has been to take the initial condition of the simulation from either expected or actual experimental conditions at 2-4 μsec after the start of the main capacitor bank discharge. The applied field, B_o, as a function of time is taken from oscilloscope traces. The simulation results without electron thermal conductivity and ion-electron and ion-ion collisions have proven effective in predicting experimental containment times. They have given good indications of expected neutron signals and ion temperatures as a function of time. They have shown that, if the plasma is stable, the containment time scales with the mirror ratio.

At this time only preliminary results are available which include the effects of ion-electron collisions and electron thermal conductivity. They show the expected cooling of the hot ions by the cold electrons.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

Pulsed θ-Pinch Reactor Burn Code*

by

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ABSTRACT

The burn code used in the energy balance calculation recently reported[1] accounted only for the burn and direct energy conversion which took place during the time in which the large constant confining field was in effect. The magnetic field takes about 20 msec to build up to this value during the adiabatic compression (stage II) heating. Likewise the field takes about 20 msec to fall to the "quenching" field value at which the plasma is just kept safely away from the first wall. It has recently come to our attention that during this rise of field just before and fall of field just after the main burn, there is enough temperature and density for both bremsstrahlung losses and thermonuclear reactions to take place. Therefore, the time range of the burn code has been extended to begin just after shock heating and extend to quenching. The forms of the rising and falling fields are tailored to agree with those expected from the magnetic energy transfer system devised by K. Thomassen.[2] The input to the burn code is now the quiescent (before shock heating) condition including filling pressure and geometrical dimensions. The simple formulas of Freidberg, Morse and Ribe[3] are used to determine the post shock heating conditions at which time the main routine of the burn code takes over and carries the problem to quenching.

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[3] J. P. Freidberg, R. L. Morse, and F. L. Ribe, Texas Symposium of the Technology of Controlled Thermonuclear Fusion Experiments, University of Texas, Austin (Nov. 1972).

* Work performed under the auspices of the U.S. Atomic Energy Commission.

Flushing and Refueling of a Pulsed Thermonuclear Reactor by Means of a Neutral Gas Layer*

by

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ABSTRACT

At the end of the burning phase in a theta pinch type reactor the reaction is quenched by suddenly reducing the confining field to a value which allows the hot central plasma to expand to a radius which is close to, but less than, that of the wall. Cold, neutral gas is rapidly introduced into the space between the central plasma and the wall by a radially inward flow through openings in the wall. A quasi-static pressure balance and heat flow is set up in the newly filled region or cooling layer allowing the central plasma to be gradually cooled by the radially outward flow of heat from the plasma to the wall. After the central plasma has lost most of its energy, it is removed by pumping.

A mathematical treatment of this problem using quasi-static pressure balance and heat flow has been worked out for the purpose of giving general estimates of the rate of heat removal for varying amounts of neutral gas being introduced. The main results of this theory are that the temperature drops approximately linearly with time and that the cooling time τ(sec) is given in terms of the average cooling layer density $n(10^{16}\ cm^{-3})$ by the formula $\tau = an^{-1.56}$ where a is determined by the plasma conditions at the end of the burn.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

TOKAMAK TRANSPORT THEORY AT OAK RIDGE NATIONAL LABORATORY*

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I: Status of Energy Balance Study for ORMAK

Recent theoretical work on low-β toroidal devices has omitted several classical processes which are, however, major factors in present experiments. Energy loss from charge exchange and ionization and the influence of impurities on Ohmic heating power input are significant and often dominant. We give a numerical calculation of particle and energy balance in a Tokamak of modest dimensions (ORMAK) and find that loss rates have not increased with scaling to larger size and to lower collision frequency. Ion losses are consistent with transport calculations.

II: Status of Advanced Tokamak Studies

We have examined the role of neutral injection in producing a controlled burning rate in a sub-critical Tokamak of larger size and find that steady states with a net power output can be obtained while avoiding secular instability.

*Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

THEORY OF NEUTRAL BEAM INJECTION INTO A TOKAMAK*

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Plasma perturbations arising from injection of a beam of energetic neutrals into a Tokamak plasma have been examined to determine if there are significant limitations on this heating method in toroidal plasmas. Specifically, the following aspects of the problem have been considered: 1) the geometry of injection including the overall beam absorption, the spatial distribution and the drift orbits (which differ significantly from the flux surfaces) of the injected particles; 2) the slowing down processes (from both a particle and kinetic viewpoint); 3) the net and distributed momentum, electric field, and current induced in the plasma dielectric constants in the neoclassical regime, ion viscosity, plasma convection, plasma instabilities, etc. For tangential injection (as in ORMAK), the most stringent limitations arise from the net and distributed momentum and current buildup that may adversely affect the toroidal plasma equilibrium and confinement. In contrast, for perpendicular injection (as in TFR) the most important limiting process is the buildup of an electric field in the poloidal direction (along a flux surface) due to the localization of the "banana" orbits of the injected trapped ions that leads to a radial $\mathbf{E} \times \mathbf{B}$ flow and hence plasma convection out of the confinement region.

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MHD STABILITY OF TOROIDAL EQUILIBRIA

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We present a new sufficient condition for the stability of toroidal closed line equilibria (e.g., bumpy tori) and of axisymmetric equilibria with shear. This condition, improving on those known previously, states that stability is guaranteed by the positivity of the lowest eigenvalue of a one-dimensional integro-differential equation closely related to the Sturm-Liouville equation. The equation has the same structure as that discussed by Bernstein, et. al., and in the limit of axisymmetry, closed field lines, and $\vec{J} \cdot \vec{B} = 0$ is identical to their equation. Thus in this limit the condition is necessary as well as sufficient. It also becomes necessary with closed field lines and reflection symmetry in the limit of low β, reducing to the full interchange condition including the "γ" term. It thus provides the first satisfactory proof that in these equilibria there is a nonzero β below which interchange stability is sufficient for absolute stability. For axisymmetric equilibria with shear the stabilizing effect of the γ term is lost, consistent with the structure of known conditions.

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